William Smalley, Springfield, Castleton, near Manchester (proposed by A. L. Cortie);
William John Thorrowgood, M.I.E.E., M.I.R.S.E., 2 Waldemar Road, Wimbledon Park, London, S.W. 19 (proposed by E. W. Maunder);
Harihar Parames Waran, M.A., Christ's College, Cambridge (proposed by A. S. Eddington); and
Dr. Jan Woltjer, Conservator at the Observatory of Leiden, Holland (proposed by W. de Sitter).

One hundred and sixty-five presents were announced as having been received since the last meeting, including, amongst others:

Drawing of Comet of 1843, presented by Miss A. Breton.
Cape of Good Hope: 2nd Fundamental Catalogue of Stars for the equinox 1900, 1912–16.
M. Dumersan: Le Zodiaque de Dendéra, presented by Mr. Inwards.
Royal Observatory, Greenwich: Catalogue of Double Stars and other publications.
San Fernando: Astrographic Chart, 42 maps.
Royal Observatory, Belgium: Astrographic Chart, 40 maps.
Hyderabad, Nizamiah Observatory: Astrographic Catalogue, vol. iv.
Helwân Observatory: Eleven slides of Nebulæ.

Note on the Path of a Ray of Light in the Einstein Relativity Theory of Gravitational Effect.

By Professor A. R. Forsyth, F.R.S., Imperial College of Science and Technology, London, S.W.

The Accurate Equation of the Path.

1. The critical equation in the Einstein theory for the gravitational deflection of a ray of light is

$$\frac{1}{\gamma}\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\phi}{dt}\right)^2 = \gamma,$$

where

$$\gamma = 1 - \frac{2m}{r}.$$

To obtain the path itself, Huygens' principle—that the time between two points is stationary for small variations of the path—is of immediate service. We have

$$\left(\frac{dt}{dr}\right)^2 = \frac{r^2}{\gamma}\left(\frac{d\phi}{dr}\right)^2 + \frac{1}{\gamma^2},$$

so that

$$t = \int\left\{\frac{r^2}{\gamma}\left(\frac{d\phi}{dr}\right)^2 + \frac{1}{\gamma^2}\right\}^{\frac{1}{2}} dr$$
$$= \int \mathrm{U}\, dr.$$

The mathematical expression of the principle is

$$\delta t = 0;$$

the customary process leads to the equation

$$\frac{d}{dr}\left(\frac{\partial U}{\partial \phi'}\right) - \frac{\partial U}{\partial \phi} = 0,$$

where ϕ' denotes $d\phi/dr$. As U does not explicitly involve ϕ, we have

$$\frac{\partial U}{\partial \phi'} = \text{constant} = E,$$

so that

$$\frac{1}{U}\frac{r^2}{\gamma}\frac{d\phi}{dr} = E;$$

and consequently

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{r^4}{E^2} - r^2 + 2mr.$$

Let D denote the apsidal distance, so that

$$\frac{D^3}{E^2} - D + 2m = 0;$$

and let the angle ϕ be measured positively from the apsidal direction, so that r and ϕ increase together. Now substitute

$$r = \frac{1}{u};$$

then

$$\begin{aligned} d\phi &= -\frac{1}{(2m)^{\frac{1}{2}}}\left(u^3 - \frac{u^2}{2m} + \frac{1}{2mE^2}\right)^{-\frac{1}{2}} du \\ &= -\frac{1}{(2m)^{\frac{1}{2}}}\left(u^3 - \frac{u^2}{2m} + \frac{D - 2m}{2mD^3}\right)^{-\frac{1}{2}} du \\ &= -\frac{1}{(2m)^{\frac{1}{2}}}\{(u-\alpha)(u-\beta)(u-\gamma)\}^{-\frac{1}{2}} du, \end{aligned}$$

where

$$\alpha = \frac{1}{4mD}\{D - 2m + (D^2 + 4Dm - 12m^2)^{\frac{1}{2}}\},$$

$$\beta = \frac{1}{D},$$

$$\gamma = \frac{1}{4mD}\{D - 2m - (D^2 + 4Dm - 12m^2)^{\frac{1}{2}}\}.$$

The quantities $1/\alpha$, $1/\beta$, $1/\gamma$ are possible apsidal distances. But $1/\alpha$ is approximately $2m$; the path would pass nearly through the centre of the sun, so that $1/\alpha$ is not an effective apsidal distance. Also, as γ is negative, $1/\gamma$ must be rejected. The only effective apsidal distance is D.

To find the value of ϕ, let

$$u = \gamma + (\beta - \gamma)\sin^2\theta,$$

$$k^2 = \frac{\beta - \gamma}{\alpha - \gamma};$$

then

$$d\phi = -\frac{1}{(2m)^{\frac{1}{2}}}\,\frac{2}{(\alpha-\gamma)^{\frac{1}{2}}}\,\frac{d\theta}{(1 - k^2\sin^2\theta)^{\frac{1}{2}}}.$$

In the path between the star and perihelion, u ranges from 0 to $\frac{1}{D}$, that is, from 0 to β; for the latter value, $\theta = \frac{1}{2}\pi$. If $\phi = \varpi$ at perihelion, we have

$$\begin{aligned}\phi - \varpi &= -2\rho\int_{\frac{1}{2}\pi}^{\theta}(1 - k^2\sin^2\theta)^{-\frac{1}{2}}d\theta \\ &= 2\rho(K - \mu),\end{aligned}$$

where

$$\rho = \{2m(\alpha - \gamma)\}^{-\frac{1}{2}},$$

$$\theta = \operatorname{am}\mu,$$

and K is the complete elliptic integral of the first kind, that is,

$$\theta = \operatorname{am}\left(K - \frac{\phi - \varpi}{2\rho}\right).$$

Thus

$$\begin{aligned}u &= \gamma + (\beta - \gamma)\sin^2\theta \\ &= \gamma + (\beta - \gamma)\operatorname{sn}^2\left(K - \frac{\phi - \varpi}{2\rho}\right),\end{aligned}$$

or

$$\frac{1}{r} = \gamma + (\beta - \gamma)\frac{1 + \operatorname{cn}\dfrac{\phi - \varpi}{\rho}}{1 + \operatorname{dn}\dfrac{\phi - \varpi}{\rho}},$$

which is the accurate (not approximate) equation of the path, on the assumption that the original critical equation is accurate.

The Path is everywhere Concave to the Sun.

2. The equation of the path is

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{r^4}{D^3}(D - 2m) - r^2 + 2mr,$$

so that

$$\frac{d^2r}{d\phi^2} = 2\frac{r^3}{D^3}(D - 2m) - r + m.$$

Hence the radius of curvature of the path, being

$$\frac{\left\{r^2+\left(\frac{dr}{d\phi}\right)^2\right\}^{\frac{3}{2}}}{r^2+2\left(\frac{dr}{d\phi}\right)^2-r\frac{d^2r}{d\phi^2}}$$

for any curve, becomes

$$\frac{r^{\frac{1}{2}}}{3m}\left\{\frac{r^4}{D^3}(D-2m)+2mr\right\}^{\frac{3}{2}}.$$

Because this is positive, the path is everywhere concave to the sun.

The smallest value of r is D. Having regard to the actual numbers, we see that the path is very slightly curved inwards to the sun at perihelion, and that, away from perihelion, the curvature is exceedingly slight.

The Co-ordinate Velocity in the Path.

3. As regards the co-ordinate velocity v, we have

$$v^2=\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\phi}{dt}\right)^2,$$

while

$$\gamma^{-1}\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\phi}{dt}\right)^2=\gamma.$$

Hence

$$v^2=\gamma^2\frac{\left(\frac{dr}{d\phi}\right)^2+r^2}{\left(\frac{dr}{d\phi}\right)^2+\gamma r^2}$$

$$=\gamma^2\left(1+\frac{2m}{D-2m}\frac{D^3}{r^3}\right),$$

and therefore

$$v=\gamma\left(1+\frac{2m}{D-2m}\frac{D^3}{r^3}\right)^{\frac{1}{2}}.$$

It is to be noted that, in obtaining this result, no approximation has been used.

At perihelion, the magnitude of the co-ordinate velocity is

$$\left(1-\frac{2m}{D}\right)^{\frac{1}{2}}.$$

The Deflection of a Ray passing from a Star to the Earth: its General Value.

4. To estimate the total deflection of a ray in passing from a star to the earth as it goes near the sun, it is convenient to divide the path into two portions, one on each side of the perihelion of the path. And, even before approximations are made, it is desirable to have a notion of the

actual numbers, though they are not introduced at this stage of the calculations.

Let R denote the radius of the sun; R_2 the distance of the earth from the sun; R_1 the distance of the star Piazzi IV, 82, from the sun.* Then we have, as working measures,

$$\frac{R_2}{R} = 2 \cdot 15 \times 10^2, \qquad \frac{R_1}{R} = 1 \cdot 63 \times 10^9.$$

Thus, for the practicably accurate range of observation, R/R_2 is a finite number though it is small, while R/R_1 is a very small number.

Consider the first portion of the path between the star and perihelion. At the star itself, let χ_1 be the angle between the solar radius vector and the tangent to the path; thus

$$\tan \chi_1 = \text{value of } r\frac{d\phi}{dr} \text{ when } r = R_1$$

$$= \frac{R_1}{\left\{\frac{R_1^4}{D^3}(D - 2m) - R_1^2 + 2mR_1\right\}^{\frac{1}{2}}}.$$

(Owing to the magnitude of R_1, the magnitude of χ_1 is negligible in comparison with practical measurements; so that the solar radius vector to the star effectively gives the direction of the emanating light. But this consideration can be deferred for the moment.)

At the star, the value of θ is given by

$$\frac{1}{R_1} = \gamma + (\beta - \gamma)\sin^2\theta_1;$$

and (again deferring the consideration of the very large value of R_1) we have

$$\phi_1 - \varpi = 2\rho(K - \mu_1),$$

where μ_1 is given by the relation

$$\theta_1 = \text{am}\,\mu_1.$$

At perihelion, the direction of the ray is perpendicular to the solar radius vector; at the star, the inclination of the ray to the solar radius vector is

$$(\phi_1 - \varpi) + \chi_1$$
$$= 2\rho(K - \mu_1) + \chi_1.$$

Hence the deflection of the ray, in passing from the star to perihelion, is

$$2\rho(K - \mu_1) + \chi_1 - \tfrac{1}{2}\pi.$$

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* *Phil. Trans.*, Ser. A, **220** (1920), p. 293. This star is selected as giving the smallest value for R_1.

I wish to take this opportunity of thanking the Astronomer Royal, Sir Frank Dyson, for his kindness in providing me with numerical data.

Similarly, on the other side of the perihelion and in the same sense, the deflection of the ray in passing from perihelion to the earth is

$$2\rho(K-\mu_2)+\chi_2-\tfrac{1}{2}\pi,$$

where

$$\tan\chi_2 = \text{value of } r\frac{d\phi}{dr} \text{ when } r = R_2$$

$$= \frac{R_2}{\left\{\frac{R_2^4}{D^3}(D-2m)-R_2^2+2mR_2\right\}^{\frac{1}{2}}},$$

where μ_2 is given by the relation

$$\theta_2 = \mathrm{am}\,\mu_2,$$

and the value of θ_2 is given by

$$\frac{1}{R_2}=\gamma+(\beta-\gamma)\sin^2\theta_2,$$

R_2 being the radial distance of the earth from the sun.

We take ϕ_2 as measured positively on the earth-side of the perihelion line. The inclination of the ray of light to the perihelion line on its arrival at the earth is

$$(\phi_2-\varpi)+\chi_2$$
$$=2\rho(K-\mu_2)+\chi_2.$$

Hence the deflection of the ray, in passing from perihelion to the earth (measured in the same sense as before) is

$$2\rho(K-\mu_2)+\chi_2-\tfrac{1}{2}\pi.$$

Consequently, the total deflection of the ray in passing from the star to the earth is

$$2\rho(K-\mu_1)+\chi_1+2\rho(K-\mu_2)+\chi_2-\pi.$$

The Approximate Expression for the Deflection of a Ray.

5. When we deal with approximations, we must select the orders. With the astronomical units in Professor Eddington's Report,

$$m = 1{\cdot}47 \text{ km.}$$

The sun's radius R is here taken to be

$$R = 695{,}500 \text{ km.};$$

and judging from the position of the selected star in one of the diagrams contained in the memoir on the 1919 eclipse, we have (roughly)

$$D = 2R.$$

Thus m/D may be taken as the small quantity of reference. Now

$$\frac{m}{R}=2{\cdot}11\times10^{-6},\quad \frac{R_2}{R}=2{\cdot}15\times10^{2},\quad \frac{R_1}{R}=1{\cdot}63\times10^{9}.$$

Having regard to the magnitude of m/D, and to the degree of accuracy that is possible in observations, we omit $(m/\mathrm{D})^2$. We keep, where they occur, R/R_2 and its powers up to 10^{-9} inclusive, that is $\left(\frac{\mathrm{D}}{\mathrm{R}_2}\right)^4$; also we keep $\frac{m}{\mathrm{R}_2}$, $\frac{m\mathrm{D}}{\mathrm{R}_2^2}$, $\frac{\mathrm{R}}{\mathrm{R}_1}$; the reason for the selection of the order will appear at the end of the calculations.

We have, up to the order retained,

$$\gamma = -\frac{1}{\mathrm{D}} + \frac{2m}{\mathrm{D}^2},$$

$$\alpha - \gamma = \frac{1}{2m} + \frac{1}{\mathrm{D}},$$

$$\beta - \gamma = \frac{2}{\mathrm{D}} - \frac{2m}{\mathrm{D}^2},$$

and so

$$k^2 = \frac{4m}{\mathrm{D}}.$$

Next,

$$\rho = 1 - \frac{m}{\mathrm{D}},$$

$$\mathrm{K} = \tfrac{1}{2}\pi\left(1 + \frac{m}{\mathrm{D}}\right).$$

Beginning with the earth-sun portion of the path, we have

$$\gamma + (\beta - \gamma)\sin^2\theta_2 = \frac{1}{\mathrm{R}_2},$$

so that

$$\cos 2\theta_2 = \frac{\frac{m}{\mathrm{D}} - \frac{\mathrm{D}}{\mathrm{R}_2}}{1 - \frac{m}{\mathrm{D}}}.$$

Now D/R_2 is greater than m/D; so, writing

$$2\theta_2 = \tfrac{1}{2}\pi + u,$$

we have

$$\sin u = \frac{\frac{\mathrm{D}}{\mathrm{R}_2} - \frac{m}{\mathrm{D}}}{1 - \frac{m}{\mathrm{D}}}$$

$$= \frac{\mathrm{D}}{\mathrm{R}_2} - m\left(\frac{1}{\mathrm{D}} - \frac{1}{\mathrm{R}_2}\right)$$

to our order; and therefore, also to the order,

$$u = \frac{\mathrm{D}}{\mathrm{R}_2} - m\left(\frac{1}{\mathrm{D}} - \frac{1}{\mathrm{R}_2}\right) + \frac{1}{6}\frac{\mathrm{D}^3}{\mathrm{R}_2^3}.$$

Thus

$$\theta_2 = \tfrac{1}{4}\pi + \frac{\mathrm{D}}{2\mathrm{R}_2} - \frac{m}{2\mathrm{D}} + \frac{m}{2\mathrm{R}_2} + \frac{1}{12}\frac{\mathrm{D}^3}{\mathrm{R}_2^3}.$$

Next,

$$\mu_2 = \int_0^{\theta_2} (1 - k^2 \sin^2 \theta)^{-\frac{1}{2}} d\theta$$
$$= \theta_2(1 + \tfrac{1}{4}k^2) - \tfrac{1}{8}k^2 \sin 2\theta_2.$$

Now

$$\sin 2\theta_2 = \cos u$$
$$= 1 - \tfrac{1}{2}u^2;$$

the term arising out of $-\frac{1}{2}u^2$ in $-\frac{1}{8}k^2 \sin 2\theta_2$ is

$$\frac{1}{4}\frac{m}{D}\frac{D^2}{R_2^{\,2}},$$

which is of the order 10^{-11}, and is rejected under the selection made; hence

$$\mu_2 = \theta_2(1 + \tfrac{1}{4}k^2) - \tfrac{1}{8}k^2$$
$$= \theta_2\left(1 + \frac{m}{D}\right) - \frac{m}{2D}$$
$$= \tfrac{1}{4}\pi - \frac{m}{D}(1 - \tfrac{1}{4}\pi) + \frac{D}{2R_2} + \frac{m}{R_2} + \frac{1}{12}\frac{D^3}{R_2^{\,3}}.$$

Again,

$$\tan \chi_2 = \frac{R_2}{\left\{ \frac{R_2^{\,4}}{D^3}(D - 2m) - R_2^{\,2} + 2mR_2 \right\}^{\frac{1}{2}}}$$
$$= \frac{D}{R_2} + \frac{m}{R_2} + \frac{D^3}{2R_2^{\,3}},$$

on neglecting terms $\frac{3}{8}\frac{D^5}{R_2^{\,5}}$ and $\frac{mD^2}{R_2^{\,3}}$ as being of too small an order; hence

$$\chi_2 = \tan \chi_2 - \tfrac{1}{3} \tan^3 \chi_2 + \ldots$$
$$= \frac{D}{R_2} + \frac{m}{R_2} + \frac{1}{6}\frac{D^3}{R_2^{\,3}};$$

up to the order.

Consequently

$$2\rho(K - \mu_2) + \chi_2$$
$$= 2\left(1 - \frac{m}{D}\right)\left[\tfrac{1}{2}\pi\left(1 + \frac{m}{D}\right) - \tfrac{1}{4}\pi + \frac{m}{D}\left(1 - \tfrac{1}{4}\pi\right) - \frac{D}{2R_2} - \frac{m}{R_2} - \frac{1}{12}\frac{D^3}{R_2^{\,3}}\right]$$
$$+ \frac{D}{R_2} + \frac{m}{R_2} + \frac{1}{6}\frac{D^3}{R_2^{\,3}}$$
$$= \tfrac{1}{2}\pi + \frac{2m}{D},$$

accurately up to the order retained.

The calculations for the portion of the path between the star and perihelion are exactly similar. The order of approximation is much

closer because R_1 is so much larger than R_2. But the result is that, even to a higher order than before,

$$2\rho(K-\mu_1)+\chi_1=\tfrac{1}{2}\pi+\frac{2m}{D}.$$

Hence, finally the total deflection of the ray, as it moves from the star, passes the sun at a perihelion distance D and comes to the earth, being equal to

$$2\rho(K-\mu_1)+\chi_1+2\rho(K-\mu_2)+\chi_2-\pi,$$

is equal to

$$\frac{4m}{D},$$

accurately up to the order 10^{-9} at least among the quantities measured.

If approximation to a higher order is desired, so that it shall be accurate up to the order 10^{-12} at least, then the whole calculation must be revised. For then, m^2/D^2 becomes significant; and an added term $mD^2/R_2{}^3$, of the same order as m^2/D^2, is introduced into the deviation. But present powers of observation seem to forbid any practical test up to that order of accuracy.

The Approximate Equation of the Path.

6. The equation to the path, approximate to the same order, is obtained simply from the accurate equation. We have

$$u=\gamma+(\beta-\gamma)\frac{1+\mathrm{cn}\dfrac{\phi-\varpi}{\rho}}{1+\mathrm{dn}\dfrac{\phi-\varpi}{\rho}}.$$

Now, when k is small, as we have

$$k^{\frac{1}{2}}=\frac{2q^{\frac{1}{4}}+2q^{\frac{9}{4}}+\ \ldots}{1+2q+\ \ldots}$$

always, we have

$$\frac{k^2}{16}=\frac{q}{(1+2q)^4},$$

approximately, that is, up to our order,

$$q=\tfrac{1}{16}k^2.$$

Let

$$\phi-\varpi=\sigma,$$

$$x=\frac{\pi\sigma}{2\rho K};$$

then

$$\mathrm{cn}\frac{\sigma}{\rho}=(1-4q\sin^2 x)\cos x,$$

$$\mathrm{dn}\frac{\sigma}{\rho}=1-8q\sin^2 x,$$

approximately. Now

$$k^2 = 4\frac{m}{D},$$

so that

$$q = \frac{m}{4D};$$

also

$$K = \tfrac{1}{2}\pi\left(1 + \frac{m}{D}\right), \quad \rho = 1 - \frac{m}{D},$$

and therefore, to the order selected,

$$\frac{\pi}{2\rho K} = 1,$$

so that

$$x = \sigma = \phi - \varpi.$$

Further

$$\gamma = -\frac{1}{D} + \frac{2m}{D^2}, \quad \beta - \gamma = \frac{2}{D}\left(1 - \frac{m}{D}\right);$$

hence

$$u = -\frac{1}{D} + \frac{2m}{D^2} + \frac{2}{D}\left(1 - \frac{m}{D}\right)\frac{1 + \cos x - \dfrac{m}{D}\sin^2 x \cos x}{2 - 2\dfrac{m}{D}\sin^2 x}$$

$$= \frac{1}{D}\cos x + \frac{m}{2D^2}(3 - 2\cos x - \cos 2x),$$

to our order: that is, the approximate equation of the path is

$$\frac{1}{r} = \frac{1}{D}\cos(\phi - \varpi) + \frac{m}{2D^2}\{3 - 2\cos(\phi - \varpi) - \cos 2(\phi - \varpi)\},$$

D being the apsidal distance.

The deviation of the path from the apsidal tangent is easily estimated by taking

$$\frac{\cos(\phi - \varpi)}{D} = \frac{1}{r_1},$$

which is the apsidal tangent; we have, at once,

$$\frac{1}{r} - \frac{1}{r_1} = \frac{m}{D^2}\left(1 - \frac{D}{r_1}\right)\left(2 + \frac{D}{r_1}\right).$$

Imperial College of Science
and Technology, S. W.:
1921 *September* 12.

On the Motion of the Perihelion of Mercury. By W. M. Smart.

§ 1. One of the outstanding discrepancies of the Newtonian Law, namely, the advance of the Perihelion of Mercury's orbit at the rate of 43″ per century, has been satisfactorily accounted for by Einstein, all the efforts of dynamical astronomers having hitherto failed to provide a solution of the problem on the basis of the Newtonian Law in relation to the known bodies of the Solar system. On the assumption that the unaccountable portion of the perturbations on Mercury might be due to an undiscovered intra-Mercurial planet ("Vulcan"), Le Verrier calculated the mass, corresponding to different heliocentric distances, of the hypothetical planet (*Annales de l'Observatoire de Paris*, tome v.; Tisserand, vol. iv., chapter 29). Other hypotheses have been considered at various times—an inner ring of planetoids, the non-sphericity of the Sun, Hall's corrected "inverse square law," and so on. I consider here only the problem of the hypothetical planet.

Recently, doubt has been thrown on the validity of the interpretation of the Michelson-Morley experiment, and even on the experiment itself (*Memorie della Società Astronomica Italiana*, nuova serie, vol. i., No. 4). Whatever justification there may be for this assault on one of the principal foundations of the Theory of Relativity, I have considered it of interest to pursue the study of the effect of a hypothetical planet further, in relation to the advance of the Perihelion of Mercury, on the Newtonian Law. The present note may thus be regarded as an extension or completion of Le Verrier's calculations, the case here considered being that of the equilateral configuration in the three-body problem. It is established that the perturbations in this particular case are considerable. Moreover, no fewer than six Trojan planets are now known, all having the same mean motion as that of Jupiter, so that the equilateral configuration seems to be a definite feature of the Solar system.

The existence of the hypothetical Vulcan, interior to Mercury, engaged the speculations of astronomers for a long time, but Vulcan still remains undiscovered. As Le Verrier showed, the mass of Vulcan, sufficient to account for the advance of 43″ per century, ranges from $\frac{1}{6}$ to $2\frac{2}{3}$ times the mass of Mercury, for heliocentric distances of Vulcan ranging from 0·27 to 0·12, the heliocentric distance of Mercury being 0·39. The comparatively large masses found by Le Verrier indicate a considerable stellar magnitude for Vulcan which would almost certainly lead to the discovery of the planet during a total eclipse of the Sun. The large perturbations resulting in the equilateral three-body problem suggest the possible existence of a Vulcan of small mass at the same heliocentric distance as Mercury. The hypothetical body in this case will sometimes be referred to, later, as the "Trojan Vulcan." The problem, then, to be considered is: "To find the mass of the Trojan Vulcan so that the advance of the Perihelion of Mercury may be accounted for on the Newtonian Law, and to obtain a rough approximation to the maximum stellar magnitude of the hypothetical planet which would afford some indication of the possibility of its discovery (or not)." If the mass turns out to be a small fraction of that of Mercury, the